

FIRST ORDER PHASE TRANSITIONS AS A SOURCE OF BLACK HOLES IN THE EARLY UNIVERSE¹

M.Yu. Khlopov^{1,2,3}, R.V. Konoplich^{1,2}, S.G. Rubin^{1,2}, A.S. Sakharov^{1,2,4}

¹ Center for CosmoParticle Physics "Cosmion"

² Moscow Engineering Physics Institute

³ Institute for Applied Mathematics

⁴ Labor für Höchenergiephysik, ETH-Hönggerberg

Abstract

A new mechanism of black hole formation in a first order phase transition is proposed. In vacuum bubble collisions the interaction of bubble walls leads to the formation of nontrivial vacuum configuration. The consequent collapse of this vacuum configuration induces the black hole formation with high probability. The primordial black holes that have been created by this way at the end of first order inflation could give essential contribution into the total density of the early Universe. The possibilities to establish some nontrivial restrictions on the inflation models with first order phase transition are discussed.

1 Introduction

At present time black holes (BH) can be created only by a gravitational collapse of compact objects with mass more than about three Solar mass¹⁾. However at the early stage of evolution of the Universe there were no limits on the mass of BH formed by several mechanisms. The simplest one is a collapse of strongly inhomogeneous regions just after the end of inflation (see e.g. ²⁾). Another possible source of BH could be a collapse of cosmic strings³⁾ that are produced in early phase transitions with symmetry breaking. The collisions of the bubble walls^{4, 5)} created at phase transitions of the first order can lead to a primordial black hole (PBH) formation.

We discuss here new mechanism of PBH production in the collision of two vacuum bubbles. The known opinion of the BH absence in such processes is based on strict conservation of the original $O(2,1)$ symmetry. Whereas there are ways to break it. Firstly, the radiation of scalar waves indicates the entropy increasing and hence the permanent breaking of the symmetry during the bubble collision. Secondly, the vacuum decay due to thermal fluctuation does not possess this symmetry from the beginning. The simplest example of a theory with bubble creation is a scalar field theory with two non degenerated vacuum states. Being stable at a classical level, the false vacuum state decays due to quantum effects, leading to a nucleation of the bubbles of true vacuum and their subsequent expansion.

¹INVITED TALK ON 10TH RUSSIAN GRAVITATIONAL CONFERENCE.

6) The potential energy of the false vacuum is converted into a kinetic energy of the bubble walls thus making them highly relativistic in a short time. The bubble expands till it collides with another one. As it was shown in ^{4, 5)} a black hole may be created in the collision of several bubbles. Our investigations show that BH can be created as well with a probability of order unity in the collisions of only two bubbles. It initiates the enormous production of BH that leads to essential cosmological consequences discussed below.

In Section 2 the evolution of the field configuration in the collisions of bubbles is discussed. The BH mass distribution is obtained in Section 3. In Section 4 cosmological consequences of the BH production in bubble collisions at the end of inflation are considered.

2 Evolution of field configuration in collisions of vacuum bubbles

Consider a theory where a probability of false vacuum decay equals Γ and difference of energy density between the false and true vacuum outside equals ρ_V . Initially bubbles are produced at rest however walls of the bubbles quickly increase their velocity up to the speed of light $v = c = 1$ because a conversion of the false vacuum energy into its kinetic ones is energetically favorable.

Let us discuss dynamics of collision of two true vacuum bubbles that have been nucleated in points $(\mathbf{r}_1, t_1), (\mathbf{r}_2, t_2)$ and which are expanding into false vacuum. Following papers ^{4, 7)} let us assume for simplicity that the horizon size is much greater than the distance between the bubbles. Just after collision mutual penetration of the walls up to the distance comparable with its width is accompanied by a significant potential energy increase ⁸⁾. Then the walls reflect and accelerate backwards. The space between them is filled by the field in the false vacuum state converting the kinetic energy of the wall back to the energy of the false vacuum state and slowdown the velocity of the walls. Meanwhile the outer area of the false vacuum is absorbed by the outer wall, which expands and accelerates outwards. Evidently, there is an instant when the central region of the false vacuum is separated. Let us note this false vacuum bag (FVB) does not possess spherical symmetry at the moment of its separation from outer walls but wall tension restores the symmetry during the first oscillation of FVB. As it was shown in ⁷⁾, the further evolution of FVB consists of several stages:

- 1) FVB grows up to the definite size D_M until the kinetic energy of its wall becomes zero;
- 2) After this moment the false vacuum bag begins to shrink up to a minimal size D^* ;
- 3) Secondary oscillation of the false vacuum bag occurs.

The process of periodical expansions and contractions leads to energy losses of FVB in the form of quanta of scalar field. It has been shown in the ^{7, 9)} that only several oscillations take place. On the other hand, important note is that the secondary oscillations might occur only if the minimal size of the FVB would be larger than its gravitational radius, $D^* > r_g$. The opposite case ($D^* < r_g$) leads to the BH creation with the mass about the mass of the FVB. As we will show later the probability of BH formation is almost unity in a wide range of parameters of theories with first order phase transitions.

3 Gravitational collapse of FVB and BH creation

Consider in more details the conditions of converting FVB into BH. The mass M of FVB can be calculated in a framework of a specific theory and can be estimated in a coordinate system K' where the colliding bubbles are nucleated simultaneously. The radius of each bubble b' in this system equals to half of their initial coordinate distance at first moment of collision. Apparently the maximum size D_M of the FVB is of the same order as the size of the bubble, since this is the only parameter of necessary dimension on such a scale: $D_M = 2b'C$. The parameter $C \simeq 1$ is obtained by numerical calculations in the framework of each theory, but its exact numerical value does not affect significantly conclusions.

One can find the mass of FVB that arises at the collision of two bubbles of radius:

$$M = \frac{4\pi}{3} (Cb')^3 \rho_V \quad (1)$$

This mass is contained in the shrinking area of false vacuum. Suppose for estimations that the minimal size of FVB is of order wall width Δ . The BH is created if minimal size of FVB is smaller than its gravitational radius. It means that at least at the condition

$$\Delta < r_g = 2GM \quad (2)$$

the FVB can be converted into BH (where G is the gravitational constant).

As an example consider a simple model with Lagrangian

$$L = \frac{1}{2} (\partial_\mu \Phi)^2 - \frac{\lambda}{8} (\Phi^2 - \Phi_0^2)^2 - \epsilon \Phi_0^3 (\Phi + \Phi_0). \quad (3)$$

In the thin wall approximation the width of the bubble wall can be expressed as $\Delta = 2(\sqrt{\lambda}\Phi_0)^{-1}$. Using (2) one can easily derive that at least FVB with mass

$$M > \frac{1}{\sqrt{\lambda}\Phi_0 G} \quad (4)$$

should be converted into BH of mass M . The last condition is valid only in case when FVB is completely contained in the cosmological horizon, namely $M_H > 1/\sqrt{\lambda}\Phi_0 G$ where the mass of the cosmological horizon at the moment of phase transition is given by $M_H \cong m_{pl}^3/\Phi_0^2$. Thus for the potential (3) at the condition $\lambda > (\Phi_0/m_{pl})^2$ the BH is formed. This condition is valid for any realistic set of parameters of theory.

The mass and velocity distribution of FVBs, supposing its mass is large enough to satisfy the inequality (2), has been found in ¹⁰⁾. This distribution can be written in the terms of dimensionless mass $\mu \equiv (\frac{\pi}{3}\Gamma)^{1/4} \left(\frac{M}{C\rho_v}\right)^{1/3}$:

$$\frac{dP}{\Gamma^{-3/4} V dv d\mu} = 64\pi \left(\frac{\pi}{3}\right)^{1/4} \mu^3 e^{\mu^4} \gamma^3 J(\mu, v), \quad (5)$$

$$J(\mu, v) = \int_{\tau}^{\infty} d\tau e^{-\tau^4}, \tau_- = \mu [1 + \gamma^2 (1 + v)].$$

The numerical integration of (5) revealed that the distribution is rather narrow. For example the number of BH with mass 30 times greater than the average one is suppressed by factor 10^5 . Average

value of the non dimensional mass is equal to $\mu = 0.32$. It allows to relate the average mass of BH and volume containing the BH at the moment of the phase transition:

$$\langle M_{BH} \rangle = \frac{C}{4} \mu^3 \rho_v \langle V_{BH} \rangle \simeq 0.012 \rho_v \langle V_{BH} \rangle. \quad (6)$$

4 First order phase transitions in the early Universe

Inflation models ended by a first order phase transition hold a dignified position in the modern cosmology of early Universe (see for example ^{11, 12}). The interest to these models is due to, that such models are able to generate the observed large-scale voids as remnants of the primordial bubbles for which the characteristic wavelengths are several tens of Mpc. ¹²). A detailed analysis of a first order phase transition in the context of extended inflation can be found in ¹³). Hereafter we will be interested only in a final stage of inflation when the phase transition is completed. Remind that a first order phase transition is considered as completed immediately after establishing of true vacuum percolation regime. Such regime is established approximately when at least one bubble per unit Hubble volume is nucleated. Accurate computation ¹³) shows that first order phase transition is successful if the following condition is valid:

$$Q \equiv \frac{4\pi}{9} \left(\frac{\Gamma}{H^4} \right)_{t_{end}} = 1. \quad (7)$$

Here Γ is the bubble nucleation rate. In the framework of first order inflation models the filling of all space by true vacuum takes place due to bubble collisions, nucleated at the final moment of exponential expansion. The collisions between such bubbles occur when they have comoving spatial dimension less or equal to the effective Hubble horizon H_{end}^{-1} at the transition epoch. If we take $H_0 = 100 h K m / \text{sec} / Mpc$ in $\Omega = 1$ Universe the comoving size of these bubbles is approximately $10^{-21} h^{-1} Mpc$. In the standard approach it believes that such bubbles are rapidly thermalized without leaving a trace in the distribution of matter and radiation. However, in the previous section it has been shown that for any realistic parameters of theory, the collision between only two bubble leads to BH creation with the probability closely to 100% . The mass of this BH is given by (see (6))

$$M_{BH} = \gamma_1 M_{bub} \quad (8)$$

where $\gamma_1 \simeq 10^{-2}$ and M_{bub} is the mass that could be contained in the bubble volume at the epoch of collision in the condition of a full thermalization of bubbles. The discovered mechanism leads to a new direct possibility of PBH creation at the epoch of reheating in first order inflation models. In standard picture PBHs are formed in the early Universe if density perturbations are sufficiently large, and the probability of PBHs formation from small post- inflation initial perturbations is suppressed exponentially. Completely different situation takes place at final epoch of first order inflation stage; namely collision between bubbles of Hubble size in percolation regime leads to PBHs formation with masses

$$M_0 = \gamma_1 M_{end}^{hor} = \frac{\gamma_1}{2} \frac{m_{pl}^2}{H_{end}}, \quad (9)$$

where M_{end}^{hor} is the mass of Hubble horizon at the end of inflation. According to (6) the initial mass fraction of this PBHs is given by $\beta_0 \approx \gamma_1/e \approx 6 \cdot 10^{-3}$. For example, for typical value of $H_{end} \approx 4 \cdot 10^{-6} m_{pl}$ the initial mass fraction β is contained in PBHs with mass $M_0 \approx 1g$.

In general the Hawking evaporation of mini BHs could give rise to a variety possible end states. It is generally assumed, that evaporation proceeds until the PBH vanishes completely ¹⁴⁾, but there are various arguments against this proposal (see e.g. ¹⁵⁾). If one supposes that BH evaporation leaves a stable relic, then it is naturally to assume that it has a mass of order $m_{rel} = km_{pl}$, where $k \simeq 1 \div 10^2$. We can investigate the consequences of PBH forming at the percolation epoch after first order inflation, supposing that the stable relic is a result of its evaporation. As it follows from our above consideration the PBHs are preferentially formed with a typical mass M_0 at a single time t_1 . Hence the total density ρ at this time is

$$\rho(t_1) = \rho_\gamma(t_1) + \rho_{PBH}(t_1) = \frac{3(1-\beta_0)}{32\pi t_1^2} m_{pl}^2 + \frac{3\beta_0}{32\pi t_1^2} m_{pl}^2 \quad (10)$$

The evaporation time scale can be written in the following form

$$\tau_{BH} = \frac{M_0^3}{g_* m_{pl}^4} \quad (11)$$

where g_* is the number of effective massless degrees of freedom.

Let us derive the density of PBH relics. There are two distinct possibilities to consider.

The Universe is still radiation dominated at τ_{BH} . This situation will be hold if the following condition is valid $\rho_{BH}(\tau_{BH}) < \rho_\gamma(\tau_{BH})$. It is possible to rewrite this condition in terms of Hubble constant at the end of inflation

$$\frac{H_{end}}{m_{pl}} > \beta_0^{5/2} g_*^{-1/2} \simeq 10^{-6} \quad (12)$$

Taking the present radiation density fraction of the Universe to be $\Omega_{\gamma_0} = 2.5 \cdot 10^{-5} h^{-2}$ (h being the Hubble constant in the units of $100 km \cdot s^{-1} Mpc^{-1}$), and using the standard values for the present time and time when the density of matter and radiation become equal, we find the contemporary densities fraction of relics

$$\Omega_{rel} \approx 10^{26} h^{-2} k \left(\frac{H_{end}}{m_{pl}} \right)^{3/2} \quad (13)$$

It is easily to see that relics overclose the Universe ($\Omega_{rel} \gg 1$) for any reasonable k and $H_{end} > 10^{-6} m_{pl}$.

The second case takes place if the Universe becomes PBHs dominated at period $t_1 < t_2 < \tau_{BH}$. This situation is realized under the condition $\rho_{BH}(t_2) < \rho_\gamma(t_2)$, which can be rewritten in the form

$$\frac{H_{end}}{m_{pl}} < 10^{-6}. \quad (14)$$

The present day relics density fraction takes the form

$$\Omega_{rel} \approx 10^{28} h^{-2} k \left(\frac{H_{end}}{m_{pl}} \right)^{3/2} \quad (15)$$

Thus the Universe is not overclosed by relics only if the following condition is valid

$$\frac{H_{end}}{m_{pl}} \leq 2 \cdot 10^{-19} h^{4/3} k^{-2/3}. \quad (16)$$

This condition implies that the masses of PBHs created at the end of inflation have to be larger then

$$M_0 \geq 10^{11} g \cdot h^{-4/3} \cdot k^{2/3}. \quad (17)$$

From the other hand there are a number of well-known cosmological and astrophysical limits¹⁶⁾ which prohibit the creation of PBHs in the mass range (17) with initial fraction of mass density closed to $\beta_0 \approx 10^{-2}$.

So one have to conclude that the effect of the false vacuum bag mechanism of PBH formation makes impossible the coexistence of stable remnants of PBH evaporation with the first order phase transitions at the end of inflation.

Acknowledgements. This work was partially performed in the framework of Section "Cosmoparticle physics" of Russian State Scientific Technological Program "Astronomy. Fundamental Space Research", International project Astrodamus, Cosmion-ETHZ and Eurocos-AMS.

References

- 1 J. R. Oppenheimer, H. Sneider, Phys. Rev. **56**, 455 (1939)
- 2 A.G.Polnarev, M.Yu.Khlopov, Sov. Phys. Usp. **145**, 369 (1985)
- 3 S. Hawking, Phys. Lett. **B231**, 237 (1989); A. Polnarev and R. Zembowicz, Phys. Rev. **D43**, 1106 (1991)
- 4 S.W.Howking, I.G.Moss, J.M.Stewart, Phys. Rev. **D26**, 2681 (1982)
- 5 I.G.Moss, Phys. Rev. **D50**, 676 (1994)
- 6 R.Watkins, M.Widrow, Nucl. Phys. **B374**, 446 (1992)
- 7 S.Coleman, Phys.Rev. **D15**, 2929 (1977)
- 8 R.V.Konoplich, Sov. Nucl. **32**, 1132 (1980)
- 9 T.I.Belova, A.E.Kudryavtsev, Physica **D32**, 18 (1988)
- 10 M.Yu.Khlopov, R.V.Konoplich, S.G.Rubin, A.S.Sakharov, hep-ph/9807343
- 11 D.La, P.J.Steinhardt, Phys. Rev. Lett. **62**, 376 (1989); R.Holman, E.W.Kolb, Y.Wang, Phys. Rev. Lett. **65**, 17 (1990); R.Holman, E.W.Kolb, S.Vadas, Y.Wang, Phys. Rev. **D43**, 3833, (1991); F.C.Adams, K.Freese, Phys. Rev. **D43**, 353 (1991); E.J.Copeland, A.R.Liddle, D.H.Lyth, E.D.Stewart, D.Wands, Phys. Rev. **D49**, 6410 (1994)
- 12 F.Occhionero, L.Amendola, Phys. Rev. **D50**, 4846 (1994); L.Amendola, C.Baccigalupi, F.Occhionero, R. Konoplich, S.Rubin, Phys. Rev. **D54**, 7199 (1996),
- 13 M.S. Turner, E.J. Weinberg and L.M. Widrow, Phys. Rev. **D46**, 2384 (1992)
- 14 S.W.Hawking, Nature (London) **248**, 30 (1974)
- 15 M.A.Markov, Quantum Gravity, Proceeding of the 2nd Seminar, Moscow, USSR, 1981, edited by M.A.Markov and P.C.West (Plenum, New York, 1984); Ya.B.Zeldovich, Quantum Gravity, Proceeding of the 2nd Seminar, Moscow, USSR, 1981, edited by M.A.Markov and P.C.West (Plenum, New York, 1984); J.D.Barrow, E.J.Copeland, A.R.Liddle, Phys. Rev. **D46**, 645 (1992); B.J.Carr, J.H.Gilbert, Phys. Rev. **D50**, 4853 (1994); S.O.Alexeyev, M.V.Pomazanov, Phys. Rev. **D55**, 2110 (1997); I.G.Dymnikova, Int. Jour. Mod. Phys. **D5**, 529 (1996)
- 16 Ya.B.Zeldovich, A.A.Starobinski, JETP Letters, **24**, 616 (1976); P.D.Naselski, Sov. Astron. J. Lett. **4**, 387 (1978); S.Miyama, K.Sato, Prog. Theor. Phys. **59**, 1012 (1978); D.Lindley, Mon. Not. R. Astron. Soc. **193**, 593 (1980); Ya.B.Zeldovich, A.A.Starobinski, M.Yu.Khlopov, V.M.Chechetkin, Sov. Astron. J. Lett. **3**, 208 (1977); T.Rothman, R.Matzaner, Astrophys. Space. Sci. **75**, 229 (1981); J.H. MacGibbon and B.J. Carr, Astrophys. J. **371**, 447 (1991)